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Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Complex Ananlysis, Probability and Linear Programming

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of statistics tables are allowed.

Module-1

1 a. With usual notations, derive Cauchy-Riemann equation in the Cartesian form. **(06 Marks)**

b. Construct an analytic function whose real part is $e^{-x}\{(x^2-y^2)\cos y + 2xy\sin y\}$ **(07 Marks)**

c. Find the analytic function f(z) given that $u - v = (x-y)(x^2+4xy+y^2)$. **(07 Marks)**

OR

2 a. If f(z) is analytic, show that
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2.$$ **(06 Marks)**

b. If $\phi + i\psi$ represents the complex potential of an electrostatic field where $\phi = x^2 - y^2 + \frac{x}{x^2+y^2}$, find ϕ and also the complex potential as a function of z. **(07 Marks)**

c. Determine the analytic function f(z) whose imaginary part is $\left(r - \frac{k^2}{r}\right)\sin\theta$. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = z^2$. **(06 Marks)**

b. Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along,

(i) The line x = 2y

(ii) The real axis upto 2 and then vertically to 2 + i. **(07 Marks)**

c. State and prove Cauchy's Integral formula. **(07 Marks)**

OR

4 a. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$ **(06 Marks)**

b. Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $(1+i)$, $(-1+i)$ and $(-1,-i)$. **(07 Marks)**

c. Evaluate $\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}\, dz$, where 'C' is the circle with $|z| = 3$. **(07 Marks)**

Module-3

5 a. A random variable X has the following probability function,

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find the value of K and also find

(i) $P(1 \le x \le 5)$

(ii) $P(x > 5)$

(iii) $P(x \le 4)$ **(06 Marks)**

b. Find the mean and standard deviation of Bionomial Distribution. **(07 Marks)**

c. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains

(i) No defective fuses.

(ii) 3 or more defective fuses. **(07 Marks)**

OR

6 a. The probability density function of a continuous random variable x is given by $P(x) = y_0 e^{-|x|}$, $-\infty < x < \infty$ find y_0. Also find mean, variance and S.D. **(06 Marks)**

b. In a certain city the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for,

(i) 10 minutes or more.

(ii) Less than 10 mins.

(iii) Between 10 and 12 minutes. **(07 Marks)**

c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D. of the distribution. Given A(0.5) = 0.19, A(1.4) = 0.42. **(07 Marks)**

Module-4

7 a. Using Simplex method, solve the LPP

Maximize : $z = 2x + 3y + z$

Subject to $x + 3y + 2z \le 11$

$x + 2y + 5z \le 19$

$3x + y + 4z \le 25$

$x, y, z \ge 0$ **(10 Marks)**

b. Solve the following LPP using Big-M method :

Maximize : $z = -2x_1 - x_2$

Subject to $3x_1 + x_2 = 3$,

$4x_1 + 3x_2 \ge 6$,

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0$ **(10 Marks)**

OR

8 a. Using Simplex method, to minimize $P = x - 3y + 2z$,

Subject to the constraints $3x - y + 2z \le 7$,

$-2x + 4y \le 12$,

$-4x + 3y + 8z \le 10$,

$x, y, z \ge 0$ **(10 Marks)**

b. Solve the following LPP using two phase method,

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$,

$x_1 + 3x_2 \leq 2$,

$x_2 \leq 4$ and

$x_1, x_2 \geq 0$ **(10 Marks)**

Module-5

9 a. Find optimal solution of the given balanced transportation problem using Vogel's method,

	A	B	C	
F_1	10	9	8	8
F_2	10	7	10	7
F_3	11	9	7	9
F_4	12	14	10	4
	10	10	8	

(10 Marks)

b. A company has three cement factories located in cities 1, 2, 3 which supply cement to four projects located in towns 1, 2, 3, 4 each plant can supply 6, 1, 10 truckloads of cement daily and daily cement requirements of the projects are respectively 7, 5, 3, 2 truckloads. The transport costs per truckload of cement (in hundreds of rupees) from each plant to each project site are as follows :

Project sites

		1	2	3	4
Factories	1	2	3	11	7
	2	1	0	6	1
	3	5	8	15	9

Determine the optimal distribution for the company so as to minimize the total transportation cost. **(10 Marks)**

OR

10 a. Obtain an initial basic solution to the following transportation problem.

To

		A	B	C	D	Availability
From	I	11	13	17	14	250
	II	16	18	14	10	300
	III	21	24	13	10	400
	Requirements	200	225	275	250	

(10 Marks)

b. A company has four machines to do four jobs. Each job can be assigned to one and only machine. The cost of each job on each machine is given in the following table :

Machines

	J \ M	M_1	M_2	M_3	M_4
Jobs	J_1	18	24	28	32
	J_2	8	13	17	19
	J_3	10	15	19	22

What are job assignments which will minimize the cost? **(10 Marks)**

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